FIND THE FIRST 3 NON-ZERO TERMS TAYLOR SERIES FOR $\tan x$ ABOUT $x = 0$

n	$f^{(n)}(x)$	$f^{n}(0)/n! * x^{n}$
0	$\tan x$	0
1	$\sec^2 x$	$1/1! * x^1$
2	$2\sec^2 x \tan x$	0
3	$4\sec^2 x \tan^2 x + 2\sec^4 x$	$2/3! * x^3$
4	$8\sec^2 x \tan^3 x + 16\sec^4 x \tan x$	0
5	$16\sec^2 x \tan^4 x + 88\sec^4 x \tan^2 x + 16\sec^6 x$	$16/5! * x^5$

$$T(x) = x + \frac{2}{3!}x^3 + \frac{\overset{2}{\cancel{16}}}{5!}x^5 + \cdots$$

$$\hookrightarrow 5 \cdot \cancel{4} \cdot 3 \cdot \cancel{2} \cdot 1$$

$$= x + \frac{1}{3}x^3 + \frac{2}{15}x^5 + \cdots$$

$$(\tan x)^{(2)} = (\tan x)'' = \frac{d^2}{dx^2}\tan x = 2\sec^2 x \tan x$$

$$(\tan x)^{(3)} = (4\sec x)(\sec x \tan x)\tan x + 2\sec^2 x(\sec^2 x)$$

$$= 4\sec^2 x \tan^2 x + 2\sec^4 x$$

$$(\tan x)^{(4)} = (8\sec x)(\sec x \tan x)\tan^2 x + 4\sec^2 x(2\tan x)(\sec^2 x)$$

$$+ 8\sec^3 x(\sec x \tan x)$$

$$= 8\sec^2 x \tan^3 x + 16\sec^4 x \tan x$$

$$(\tan x)^{(5)} = (16\sec x)(\sec x \tan x)\tan^3 x + 8\sec^2 x(3\tan^2 x)(\sec^2 x)$$

$$+ (64\sec^3 x)(\sec x \tan x)\tan x + (16\sec^4 x)\sec^2 x$$

$$= 16\sec^2 x \tan^4 x + 88\sec^4 x \tan^2 x + 16\sec^6 x$$

P. 768 OF TEXTBOOK – TABLE OF COMMON TAYLOR SERIES ABOUT $x=0$ (IE. MACLAURIN SERIES)

REMINDER FROM PRECALC

BINOMIAL THEOREM

IF $p \in \mathbb{Z}^*$ ← non-neg

THEN $(x+y)^p = \sum_{n=0}^{p} \binom{p}{n} x^{p-n} y^n$

$$= \binom{p}{0}x^p + \binom{p}{1}x^{p-1}y + \binom{p}{2}x^{p-2}y^2 + \cdots + \binom{p}{p-1}xy^{p-1} + \binom{p}{p}y^p$$

$$\binom{p}{n} = \frac{p!}{n!\,(p-n)!} = \frac{p(p-1)(p-2)\cdots(p-n+1)\cancel{(p-n)!}}{n!\,\cancel{(p-n)!}}$$

"p CHOOSE n"

$$\binom{p}{n} = \frac{p(p-1)(p-2)\cdots(p-n+1)}{n!}$$ ← SAME # OF FACTORS IN NUMER AS IN EXPANDED DENOM

$$\text{eg. } \binom{\frac{1}{2}}{3} = \frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}-2)\cancel{\cdots\underbrace{(\tfrac{1}{2}-3+1)}_{\frac{1}{2}-2}}}{3!} = \frac{\frac{1}{2}(+\frac{1}{2})(+\frac{\cancel{3}}{2})}{\cancel{3}\cdot 2\cdot 1} = \frac{1}{16}$$

$$\binom{p}{n} = \frac{p(p-1)(p-2)\cdots(p-n+1)}{n!}$$

$$\binom{p}{0} = \frac{p!}{0!\,(p-0)!} = \frac{p!}{1\cdot p!} = 1$$

$$\binom{p}{1} = \frac{p}{1} = p$$

$$\binom{p}{n+1} = \frac{p(p-1)(p-2)\cdots(p-n+1)(p-n)}{\cancel{(n+1)!}} = \frac{p-n}{n+1}\binom{p}{n}$$

$$(n+1)\,n!$$

$n=1 \rightarrow$
$n+1=2 \rightarrow$ $$\binom{p}{2} = \frac{p-1}{1+1}\binom{p}{1} = \frac{p-1}{2}\,p = \frac{p(p-1)}{2}$$

CHECK: $\frac{p(p-1)}{2\cdot 1} = \frac{p(p-1)}{2!}$

$n+1=4$
$n=3$ $\rightarrow$ $$\binom{\frac{1}{2}}{4} = \frac{\frac{1}{2}-3}{4}\binom{\frac{1}{2}}{3} = \frac{-\frac{5}{2}}{4}\,\frac{1}{16} = \frac{-5}{128}$$

FIND THE TAYLOR SERIES FOR $(1+x)^p$ ABOUT $x=0$

n	$f^{(n)}(x)$	$f^{(n)}(0)/n! * x^n$	
0	$(1+x)^p$	$1/0! * x^0$	$\binom{p}{0}x^0$
1	$p(1+x)^{p-1}$	$p/1! * x^1$	$\binom{p}{1}x^1$
2	$p(p-1)(1+x)^{p-2}$	$p(p-1)/2! * x^2$	$\binom{p}{2}x^2$
3	$p(p-1)(p-2)(1+x)^{p-3}$	$p(p-1)(p-2)/3! * x^3$	$\binom{p}{3}x^3$
4	$p(p-1)(p-2)(p-3)(1+x)^{p-4}$	$p(p-1)(p-2)(p-3)/4! * x^4$	$\binom{p}{4}x^4$

$$(1+x)^p \quad T(x) = \sum_{n=0}^{\infty} \binom{p}{n} x^n$$

NOTE: PRECALC BINOMIAL TH'M: $(x+y)^p = \sum_{n=0}^{p} \binom{p}{n} x^{p-n} y^n$

$$(1+x)^p = \sum_{n=0}^{p} \binom{p}{n} 1^{p-n} x^n = \sum_{n=0}^{p} \binom{p}{n} x^n$$

IF $p \in \mathbb{Z}^*$

AND $n > p$ $\qquad \binom{p}{n} = \dfrac{p(p-1)(p-2) \quad (p-n+1)}{n!}$ $\swarrow$ LAST FACTOR IS EITHER 0 OR NEGATIVE

$\uparrow$ ONE OF THE PREVIOUS FACTORS WAS 0

$= 0$

$n > p \rightarrow 0 > p - n$

$1 > p - n + 1$

$p - n + 1 < 1$

BINOMIAL TH'M

$$\boxed{(1+x)^p = \sum_{n=0}^{\infty} \binom{p}{n} x^p}$$